Broader Collaborators

**Lockheed Martin**
Greg Tallant, Peter Stanfill, and Steve Adachi

**ETH**
Matthias Troyer and Troels Ronnow

**Google**
Sergio Boixo

**UCL**
Paul Warburton and Walter Vinci
Outline

D-Wave Machine

Research Results to Date

Applications

Summary
The End of Dennard Scaling

Data collected by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, C. Batten
Need More Capability?

Exploit a New Phenomenon
D-Wave Quantum Annealer

Massive Scaling
Tianhe-2 (3M cores)

Application Specific Systems
D.E. Shaw Research Anton
Adiabatic Quantum Annealing

Problem: find the ground state of

$$H_{\text{Ising}} = \sum_j h_j \sigma_j^z + \sum_{(i,j)\in E} J_{ij} \sigma_i^z \sigma_j^z$$

Shown by Barahona (1982) to be NP-hard in 2D, $J_{ij} = \pm$, $h_j \neq 0$.

Use adiabatic interpolation from transverse field (Farhi et al., 2000)

$$H(t) = A(t) \sum_j \sigma_j^x + B(t) H_{\text{Ising}}$$

$t \in [0, t_f]$  

Program \{h_i\}, \{J_{ij}\}

Graph Embedding implemented on DW-1 via Chimera graph retains NP-hardness (V. Choi, 2010)
D-Wave System Collage
Eight Qubit Unit Cell

Images courtesy D-Wave
Chimera Graph Topology

The topology of the D-Wave Two at the USC – Lockheed Martin Quantum Computing Center.

503 of 512 qubits calibrated and mapped.
D-Wave’s Version of “Moore’s Law”

Number of Qubits

Date

1-Jan-03 5-Jun-04 9-Nov-05 15-Apr-07 18-Sep-08 22-Feb-10 29-Jul-11 1-Jan-13

4-qubit Calypso

28-qubit Leda

128-qubit Rainier

512-qubit Vesuvius

16-qubit Europa

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Quantum Signature: Degenerate Ising Hamiltonian

\[ H_{\text{Ising}} = \sum_j h_j \sigma_j^z + \sum_{(j,k) \in E} J_{jk} \sigma_j^z \sigma_k^z \]

\[ h_j = -1, \ h_j = 1, \ J_{ik} = -1 \]

17-fold degenerate ground space:

\[ \pm 1 \ \pm 1 \ \pm 1 \ \pm 1 \ 1 \ 1 \ 1 \ 1 \ 1 \]

\[ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1 \]
Simulated Annealing At Several Speeds

Probability vs. temperature for different speeds

E1 (spins down) is always more probable
Simulated Quantum Annealing

Gap 1.5 GHz
(Temp: 0.35 GHz)
D-Wave One Experiments

144 embeddings

Quantum Signature: the isolated state is suppressed

Scalable Quantum Signature

Witness for Entanglement

Collaboration with D-Wave
Only they can take the tunneling microscopy measurements needed
Allows us to infer the value of an entanglement witness

Lanting, et.al, PRX 4, 021001 2014
Performance of Classical Exact Solvers
Quantum Annealing vs. Simulated Annealing

Benchmarks with 2-range glasses without fields

D-Wave 2 vs. Eight-core Pentium (USC & ETH)
Quantum Annealing vs. Simulated Annealing

Benchmarks with 2-range glasses without fields

D-Wave 2 vs. Nvidia Kepler GPU (USC & ETH)
Hard problems motivated by satisfiability

Random walk to create frustrated loops that respect a planted solution

Frustration

Itay Hen, Performance of D-Wave Two on Problems with Planted Solutions, AQC 2014
Hard problems for multiple heuristics

- "at least once with 99% chance" - a comparison.
- Universal peak in hardness.
Performance on hard problems

- “average time to solution” scaling results.

Graphs showing the relationship between the number of loops and the number of qubits, with average time to solution on the y-axis and the square root of the number of qubits on the x-axis.

- D-Wave
- Selby
- SA
- SSSV
Speedup relative to what?

Better heuristics?
Selby motivated by earlier comparisons with simulated annealing

Multiple Xeon cores?
SAT solvers often don’t scale well
MPI overheads are O(us).

Not the only way to use CMOS
FPGA circuits (Victor Martin-Major’s earlier Janus talk)
ASICS ala Anton (Mark Moraes’s talk)
Full custom circuits

“D-Wave problem”
Best case for D-Wave
Not at all clear that this will extend to real applications
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## What Problems Might be Amenable?

<table>
<thead>
<tr>
<th>Problem</th>
<th>Application</th>
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</thead>
<tbody>
<tr>
<td>Traveling salesman</td>
<td>Logistics, vehicle routing</td>
</tr>
<tr>
<td>Minimum Steiner tree</td>
<td>Circuit layout, network design</td>
</tr>
<tr>
<td>Graph coloring</td>
<td>Scheduling, register allocation</td>
</tr>
<tr>
<td>MAX-CLIQUE</td>
<td>Social networks, bioinformatics</td>
</tr>
<tr>
<td>QUBO</td>
<td>Machine learning</td>
</tr>
<tr>
<td>Integer Linear Programming</td>
<td>Natural language processing</td>
</tr>
<tr>
<td>Sub-graph isomorphism</td>
<td>Cheminformatics, drug discovery</td>
</tr>
<tr>
<td>Motion planning</td>
<td>Robotics</td>
</tr>
<tr>
<td>MAX-2SAT</td>
<td>Artificial intelligence</td>
</tr>
</tbody>
</table>

NP-complete problems from Wikipedia
On-Going Applications Research

Verification and Validation
Model checking for DARPA
Cyber physical system V&V at Lockheed Martin

Optimization of System Design
Collaboration with Lockheed Martin

Image Processing
Image registration at D-Wave
Image recognition at Google
Organic photovoltaic triage at Harvard

Missile Defense
Closely-spaced objects at Aerospace
Model Checking

- Given a:
  - Finite transition system $M$
  - A temporal property $p$
- The model checking problem:
  - Does $M$ satisfy $p$?

- Counter-example guided abstraction refinement
  - Identify combinatorial optimization components of classical algorithms and apply AQO
Nested Dissection Reordering

**Best reordering algorithm for many linear systems**
Widely used in practice today
Available in packages such as Metis and Scotch

**Basic ND algorithm**
Find a small separator to partition the matrix into two halves
In general, this is an NP-complete problem
Refine the separator, “straightening” it
minimize separator length
balance the two subgraphs
Recursively partition the remaining subgraphs
Many Partitioning Heuristics Available

Ones I know used

Level Sets
George and Liu
Ashcraft and Grimes

Moment of Inertia

Fiduccia-Mattheyses

Spectral Bisection
Fiedler vectors

Ones I don’t know of

Simulated Annealing

Quantum Annealing
Ising Models for NP-hard Problems

Graph bisection can be described as an Ising spin glass
J Phys A19 1605 (1986)

Since then, Ising models for many other problems have been found too
arXiv: 1302.5843

The general goal: find a cost function $H$ (Hamiltonian) whose solutions correspond to the hard problem
Create energy penalties for suboptimal solutions
Ising model for Graph Partitioning

\[ H = A \left( \sum_{i \in V} \sigma_i \right)^2 + B \sum_{ij \in E} \frac{1 - \sigma_i \sigma_j}{2} \]

- Penalize configurations that don’t bisect
- Add energy penalty \( A \) to configurations that don’t bisect
- Add energy penalty \( B \) to edges in the separator
- Find an edge separator

Partition the graph into two domains: spin up or down
Add energy penalty \( A \) to configurations that don’t bisect
Add energy penalty \( B \) to edges in the separator
Finds an edge separator
Toy problem

Solutions $^T$

\[
\begin{array}{c}
- - - \\
- - + \\
- + - \\
- + + \\
+ - - \\
+ - + \\
+ + - \\
+ + + \\
\end{array}
\]

$H$

\[
\begin{array}{c}
1 0 1 \\
0 1 0 \\
1 0 1 \\
\end{array}
\]

All possible solutions

\[
\begin{array}{c}
- - - - + + + + \\
- - + + - - + + \\
- + - + - + - + \\
\end{array}
\]

Energy

\[
\begin{array}{c}
5 \\
1 \\
5 \\
1 \\
1 \\
5 \\
\end{array}
\]
Direct embedding of Ising Model to Chimera Graph

Use strong couplings between qubits to create virtual nodes with full connectivity

Strong coupling forces coupled qubits into the same state for low energy solutions

**Pros:** Allows embedding of any graph topology with N nodes, no classical post-processing necessary

**Cons:** Limited by number of qubits, Mapping problem to chimera graph for large N is NP-Hard, Each additional strong coupling reduces effect of weaker couplings
Test Results: Fully Embedded 10 Node Line Graph

Ideal Partition appears as the lowest energy solution state

<table>
<thead>
<tr>
<th>E (-30)</th>
<th>N</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.40E-03</td>
<td>11</td>
<td>[1 1 1 1 1 -1 -1 -1 -1 -1]</td>
</tr>
<tr>
<td>-7.40E-03</td>
<td>1</td>
<td>[-1 -1 -1 -1 -1 1 1 1 1 1]</td>
</tr>
<tr>
<td>-6.40E-03</td>
<td>29</td>
<td>[1 1 1 1 -1 -1 -1 -1 -1 -1]</td>
</tr>
<tr>
<td>-6.40E-03</td>
<td>3</td>
<td>[1 1 1 1 1 -1 -1 -1 -1 -1]</td>
</tr>
<tr>
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<td>1</td>
<td>[-1 -1 -1 -1 1 1 1 1 1 1]</td>
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<tr>
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</tr>
<tr>
<td>-6.00E-03</td>
<td>32</td>
<td>[-1 1 1 1 1 1 -1 -1 -1 -1]</td>
</tr>
</tbody>
</table>

Dense Hamiltonian Coupling Factors:
Strong: -1
Adjacent: .0002
Weak: .0005

Small differences in solution state energies due to large relative weight of strong couplings

Future Quantum Annealers with higher connectivity and more qubits may make Direct Embedding more realistic
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After little over a decade, adiabatic quantum annealing is moving from theory to practice.

Today’s D-Wave system raises a variety of research questions that USC, Lockheed Martin, and our colleagues are jointly investigating.

Future D-Wave systems might soon be the most powerful on Earth for a range of problems.

What those might be, and how to program them, are still open research problems.
Open Research Problems

Why does the D-Wave even work?
Its an open system

How much quantum speedup will there be?
Any? If so, on what problems?

What applications will it ultimately solve?
We’ve had half a century to find competing heuristics

How should you program it?
Specifically excluded from recent research programs

What should the topology be?
Reduce critical scaling limitation

Other adiabatic quantum systems will face these.
These questions are all bigger than just D-Wave
Back Up
Case Study: 4-city Traveling Salesman (LM 2011)

- 4 city directed Traveling Salesman Problem
- 12 logical qubits (1 per directed link)
  - Embeddable on 92-qubit processor
- Found optimal solution in 10% of iterations

Slide Courtesy of Steve Adachi, Lockheed Martin
Case Study: Protein Folding (Harvard/D-Wave)

- Simplified 2-dim lattice model of protein folding
- Modeled a 6 amino acid chain

- Model has 18 logical qubits
- Embedding took 81 physical qubits

- Found correct ground state on 13 of 10,000 measurements! (0.13%)


Slide Courtesy of Steve Adachi, Lockheed Martin